his question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | | |

. No. of Question Paper : **92**

Inique Paper Code : **32351303** **I**

lame of the Paper : **C-7 Multivariate Calculus**

lame of the Course : **B.Sc. (Hons.) Mathematics**

emester : **III**

Duration : **3 Hours** **Maximum Marks : 75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All sections are compulsory.

All questions carry equal marks.

Section I

Attempt any *six* questions from this section.

1. Let f be the function defined by $f(x, y) = \dfrac{x^2 + 2y^2}{x^2 + y^2}$ for $(x, y) \neq (0, 0)$.

(*a*) Find $\lim\limits_{(x, y) \to (2, 1)} f(x, y)$.

(*b*) Prove that f has no limit at (0, 0).

P.T.O.

2. The temperature at the point (x, y) on a given metal pl
in the xy-plane is determined according to the form
$T(x, y) = x^3 + 2xy^2 + y$ degrees. Compute the rate at wh
the temperature changes with distance if we start at (2,
and move :

(a) parallel to the vector **j**. 5

(b) parallel to the vector **i**. 6

3. The Company sells two brands X and Y of a commerc
soap, in thousand-pound units. If x units of brand X a
y units of brand Y are sold, the unit price for brand X
$p(x) = 4{,}000 - 500x$ and for brand Y is $q(y) = 3{,}000 - 450$

(a) Find the total revenue R in terms of p and q.

(b) Suppose the brand X sells for \$ 500 per unit and bra 8
Y sells for \$ 750 per unit. Estimate the change in to
revenue if the unit prices are increased by \$ 20 for bra
X and \$ 18 for brand Y.

f

$$w = f\left(\frac{r-s}{s}\right),$$

show that

$$r\frac{\partial w}{\partial r} + s\frac{\partial w}{\partial s} = 0.$$

Find the directional derivative of $f(x, y) = e^{x^2y^2}$ at P(1, –1) in the direction toward Q(2, 3).

Find the absolute extrema of $f(x, y) = 2\sin x + 5\cos y$ in the rectangular region with vertices (0, 0), (2, 0), (2, 5) and (0, 5).

Let $\mathbf{R} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$ and $r = \|\mathbf{R}\|$, evaluate $\operatorname{div}\left(\frac{1}{r^3}\mathbf{R}\right)$.

Section II

Attempt any *five* questions from this section.

By using iterated integral, compute

$$\iint_R x\sqrt{1-x^2}\,e^{3y}\,dA,$$

where R is the rectangle $0 \le x \le 1$, $0 \le y \le 2$.

P.T.O.

9. Evaluate the double integral :

$$\iint_D \frac{dA}{y^2+1},$$

where D is the triangular region bounded by $y = -x$ an
14.
$y = 2$.

10. Evaluate the double integral

$$\int_0^2 \int_y^{\sqrt{8-y^2}} \frac{1}{\sqrt{1+x^2+y^2}}\, dx\, dy$$

by converting to polar co-ordinates.

11. Find the volume of the tetrahedron T bounded by the pla $2x + y + 3z = 6$ and the co-ordinates plane $x = 0$, $y = 0$ ar $z = 0$.

12. Find the volume of the solid D bounded by the parabolo $z = 1 - 4\left(x^2 + y^2\right)$ and the xy-plane.

13. Evaluate

$$\iint_D (x+y)^5 (x-y)^2\, dy\, dx$$

by using change of variable $u = x + y$ and $v = x -$ where D is the region in the xy-plane which is bounded the co-ordinate axes and the line $x + y = 1$.

Section III

Attempt any *four* questions from this section.

14. Evaluate the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{R},$$

where

$$\mathbf{F} = \frac{x}{\sqrt{x^2 + y^2}} i - \frac{y}{\sqrt{x^2 + y^2}} j$$

and C is the quarter circle path $x^2 + y^2 = a^2$, traversed from $(a, 0)$ to $(0, a)$.

15. Show that the vector field

$$\mathbf{F}(x, y, z) = \langle \sin z, -z \sin y, x \cos z + \cos y \rangle$$

is conservative and evaluate

$$\int_C \mathbf{F} \cdot d\mathbf{R}$$

for any piecewise smooth path joining A(1, 0, −1) to B(0, −1, 1).

P.T.O.

16. Use Green's theorem, to find the work done by the force field

$$\mathbf{F}(x, y) = (3y - 4x)\mathbf{i} + (4x - y)\mathbf{j}$$

when an object moves once counterclockwise around the ellipse $4x^2 + y^2 = 4$.

17. Use Stokes' theorem, to evaluate the line integral

$$\oint_C (3y\,dx + 2z\,dy - 5x\,dz)$$

where C is the intersection of the xy-plane and the hemisphere

$$z = \sqrt{1 - x^2 - y^2},$$

traversed counterclockwise as viewed from above.

18. Evaluate

$$\iint_S (\mathbf{F}.\mathbf{N})\,dS,$$

where $F = x^2\mathbf{i} + xy\mathbf{j} + x^3y^3\mathbf{k}$ and S is the surface of the tetrahedron bounded by the plane $x + y + z = 1$ and the coordinate planes, with outward unit normal vector **N**.